

p -BRANE and D -BRANE ACTIONS

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We consider the actions for ten-dimensional p -branes and D -branes in an arbitrary curved background and discuss some of their properties. We comment on how the $SL(2, R)$ duality symmetry acts on the five-brane actions.

1 Introduction

The original classification of super p -branes was based on the assumption that the embedding coordinates, in a physical gauge, form worldvolume scalar multiplets¹. A classification of such scalar multiplets with T (transverse) scalar degrees of freedom in $p + 1$ dimensions leads to the following table:

Table 1: Scalar multiplets with T scalars in $p + 1$ dimensions.

| $p + 1$ | T | T | T | T |
|---------|-----|-----|-----|-----|
| 1 | 1 | 2 | 4 | 8 |
| 2 | 1 | 2 | 4 | 8 |
| 3 | 1 | 2 | 4 | 8 |
| 4 | | 2 | 4 | |
| 5 | | | 4 | |
| 6 | | | 4 | |

Each scalar multiplet corresponds to a p -brane in d target spacetime dimensions with

$$d = (p + 1) + T. \quad (1)$$

Here the target space has been divided into $p + 1$ worldvolume and T transverse directions. The Table describes 16 p -branes with $p \leq 5$ and $d \leq 11$.

The corresponding p -brane actions consist of a kinetic term and a Wess-Zumino (WZ) term. The kinetic term is given by

$$S_{\text{kin}}^{(p)} = \int d^{p+1}\xi \sqrt{|g|}, \quad (2)$$

where g is the determinant of the induced metric. The WZ term is given by the pull-back of a $(p + 1)$ -form potential. For the case of interest (10 dimensions)

these are a two-index Neveu-Schwarz/Neveu-Schwarz (NS/NS) tensor $B^{(1)}$ (Type I, IIA or IIB one-brane) and the dual six-index tensor $\tilde{B}_I^{(1)}$ (Type I five-brane):

$$S_{\text{WZ}}^{(1)} = \int d^2\xi B^{(1)}, \quad (3)$$

$$S_{\text{WZ}}^{(5)} = \int d^6\xi \tilde{B}_I^{(1)}, \quad (4)$$

where $d\tilde{B}_I^{(1)} = *dB^{(1)}$. Note that the action (3) is the same independent of whether the one-brane is propagating in a $N = 1$, IIA or IIB supergravity background.

2 D -Branes

Recently, a new class of extended objects has been introduced, called Dirichlet branes or Dp -branes². They are described by embedding coordinates that, in a physical gauge, form worldvolume vector multiplets. A classification of all vector multiplets with T (transverse) scalars in $p + 1$ dimensions is given by the table below.

Table 2: Vector multiplets with T scalar degrees of freedom in $p + 1$ dimensions.

| $p + 1$ | T | T | T | T |
|---------|-----|-----|-----|-----|
| 1 | 2 | 3 | 5 | 9 |
| 2 | 1 | 2 | 4 | 8 |
| 3 | 0 | 1 | 3 | 7 |
| 4 | | 0 | 2 | 6 |
| 5 | | | 1 | 5 |
| 6 | | | 0 | 4 |
| 7 | | | | 3 |
| 8 | | | | 2 |
| 9 | | | | 1 |
| 10 | | | | 0 |

In the case of ten dimensions, this leads to Dp -branes for $0 \leq p \leq 9$. The kinetic term of these Dp -branes is given by the following Born-Infeld type action:

$$S_{\text{kin}}^{(Dp)} = \int d^{p+1}\xi \, e^{-\phi} \sqrt{|\det(g_{ij} + \mathcal{F}_{ij})|}, \quad (5)$$

where g_{ij} is the embedding metric and $\mathcal{F} = 2dV - B^{(1)}$ is the curvature of the worldvolume gauge field V . There is also a WZ term which describes the coupling of the Ramond–Ramond (RR) fields to the Dp -brane. In the case of the $D1$ -brane and $D5$ -brane³ these WZ terms are given by ^a

$$S_{\text{WZ}}^{(D1)} = \int d^2\xi \, [B^{(2)} + \ell\mathcal{F}], \quad (6)$$

$$S_{\text{WZ}}^{(D5)} = \int d^6\xi \, [\tilde{B}_{\text{IIB}}^{(2)} + \frac{1}{4}(B^{(1)})^2 B^{(2)} + D\mathcal{F} + \frac{3}{4}B^{(1)}B^{(2)}\mathcal{F} + \frac{3}{4}B^{(2)}\mathcal{F}^2 + \ell\mathcal{F}^3], \quad (7)$$

where $\tilde{B}_{\text{IIB}}^{(i)}$ ($i = 1, 2$) is the IIB-dual of $B^{(i)}$:

$$d\tilde{B}_{\text{IIB}}^{(i)} + \epsilon^{ij}DdB^{(j)} - \frac{1}{4}\epsilon^{ij}\epsilon^{kl}B^{(j)}B^{(k)}dB^{(l)} = *dB^{(i)}. \quad (8)$$

The WZ terms of the Type IIB 1-brane in (3) and the $D1$ -brane in (6), together with their kinetic terms, are related to each other via a worldvolume Poincaré duality that replaces the worldvolume vector V by a constant c . In this process the background fields get replaced by their S -duals, e.g. $B^{(1)}$ gets replaced by $B^{(2)} - cB^{(1)}$, etc. Together with a shift symmetry of the scalar ℓ , this leads to a realization of the $SL(2, R)$ duality group on the Type IIB 1-brane actions (3), (6)⁵.

The situation differs for the WZ terms of the 5-brane (4) and the $D5$ -brane (7). The reason for this is that the 5-brane (4) is a Type I 5-brane with $4 + 4$ worldvolume degrees of freedom whereas the $D5$ -brane is a Type IIB 5-brane with $8 + 8$ degrees of freedom. To realize the $SL(2, R)$ duality symmetry on the $D5$ -brane action, we need a second Type IIB 5-brane action. For this purpose we cannot consider the direct dimensional reduction of the eleven-dimensional five-brane since this reduction leads to a Type IIA five-brane that naturally couples to the Type IIA dual potential $\tilde{B}_{\text{IIA}}^{(1)}$:

$$d\tilde{B}_{\text{IIA}}^{(1)} - \frac{105}{4}CdC - 7A^{(1)}G(\tilde{C}) = *dB^{(1)}. \quad (9)$$

Here $G(\tilde{C})$ is the curvature of the dual 5-form potential \tilde{C} defined in⁶.

^a All terms are fixed by gauge invariance except for the last terms whose coefficient can be determined via T -duality⁴.

In analogy with the the case of the one-brane described above we expect that under a worldvolume Poincaré duality transformation the $D5$ -brane gets converted into another Type IIB 5-brane where the vector field V is replaced by a 3-form gauge field W . Due to the presence of higher-order \mathcal{F} terms in the WZ term (7) this duality transformation will be nonlinear and difficult to perform explicitly. Perhaps a better approach, based on an analogy with a formulation of the $D4$ -brane action⁶, is to work with a 1-form V and a 3-form W at the same time, and to eliminate one or the other at the level of the field equations via a curvature constraint. We find that the gauge transformations and curvature for W are given by

$$\begin{aligned}\delta W &= \rho + 3\Sigma^{(2)}dV, \\ \mathcal{H} &= dW - D - \frac{3}{4}B^{(1)}B^{(2)} - \frac{3}{2}B^{(2)}\mathcal{F}.\end{aligned}\tag{10}$$

Our next task is to construct a self-dual action such that the constraint $\mathcal{H} = e^{-\phi} *\mathcal{F}$ can be imposed consistently. It seems that this is indeed possible. It would be interesting to see how the $SL(2, R)$ symmetry works in the context of this selfdual action.

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